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1D volume metamaterial derived from LH parallel strips

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Abstract – A left-handed parallel strip transmission line, the simplest example of a left-handed medium, is analyzed by various methods in this paper. The design, analysis, fabrication and measurement of a new 1D volume metamaterial is reported. The new medium is made up of left-handed parallel strips. The series capacitors are realized by short circuited parallel strip stubs, and the conducting shunt pins are shunt inductors. This metamaterial allows the propagation of a left-handed wave in the frequency band from 4.75 to 5.65 GHz, much wider than the frequency band achievable in the case of split-ring resonators. The simple equivalent circuit of the metamaterial is derived.

Index terms – dispersion characteristic, metamaterial, left-handed medium, parallel strips.

I. Introduction

Standard natural materials have positive permittivity and permeability, i.e., a positive refractive index. Huge opportunities to obtain materials with various refractive index values assigned to various applications have opened up with the availability of metamaterials [1-3]. A medium with negative permittivity and permeability can thus be obtained. Metamaterials are also referred to as left-handed (LH) materials, due to the orientation of the vectors of the electric and magnetic field and the propagating vector of the traveling wave.

LH transmission lines and materials are designed as structures with periodically inserted inclusions. If the dimensions of an inclusion, i.e., a unit cell of this periodic structure, are infinitesimal, then this medium can be treated as continuous. Otherwise the medium must be treated as a periodic structure and the dispersion characteristic is determined with the use of the Floquet theorem. The dispersion characteristic can also be calculated numerically using a professional field solver, e.g., CST Microwave Studio. The application of these methods is shown in this paper, using LH parallel strips – the simplest LH medium.

We have proposed, designed and fabricated a new volume left-handed metamaterial. Its structure duplicates the concept of left-handed parallel strips cut by series capacitors and shunted by shunt inductors. The structure shows typical left-handed behaviour in a frequency band wider than the band of structures based on resonant elements [1, 3]. The wavelength of the excited LH wave increases with increasing frequency. This, together with the animation of the simulated electromagnetic field and the negative refraction of the wave on the boundary between this material and air, confirms the left-handed character of the medium. The transmission and dispersion characteristics of this medium were calculated and measured. They fit each other very well. The structure represents a volume metamaterial illuminated by a plane electromagnetic wave. However, the plane wave inside the material propagates as in a 1D transmission line.

II. Concept of an LH medium and its dispersion characteristic

First, let us discuss the concept of an LH medium, its characterization by the equivalent circuit, the dispersion characteristics, and the ways in which they can be determined.

A medium transmitting an electromagnetic wave can be modeled by a homogeneous transmission line. The equivalent circuit of one cell of a general transmission line consists of series impedance Z_S and shunt admittance Z_P taken per unit length. The length d of this cell must be infinitesimally short. A standard transmission line has an inductive series impedance representing the stored energy of the magnetic field and a capacitive shunt admittance representing the stored energy of the electric field. Thus, for a lossless line we have $Z_S = j\omega L_R$, $Y_p = j\omega C_R$, and the well known formulae for the characteristic impedance Z_0 and the propagation constant $\gamma = j\beta$ of this transmission line are given, e.g. in [4], and β is the phase constant. Now let us consider the dual case with exchanged positions of the capacitor and the inductor (Fig. 1a). In this way we have changed the original *L-C* low-pass structure into a *C-L* high-

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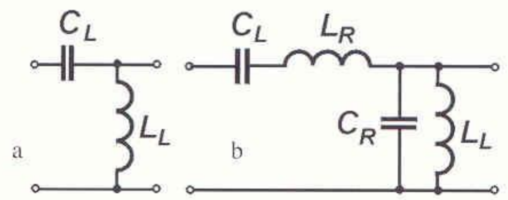

Fig. 1. The equivalent circuit of an ideal left-handed line (a), and a real LH transmission line (b). All circuit elements are taken per unit length.

pass structure. For a lossless line now $Z_s = 1/(\mathrm{j}\omega C_L)$ and $Y_p = 1/(\mathrm{j}\omega L_L)$ and we get

$$(1)\quad Z_0 = \sqrt{\frac{L_L}{C_L}}$$

$$(2)\quad \beta = -\frac{1}{\omega\sqrt{L_L C_L}}$$

The group velocity has the opposite direction to the phase velocity. This is a feature of a backward wave known as a left-handed wave. The phase propagates in the direction opposite to the flow of the transmitted power. The value of the phase constant decreases with frequency, which means that the wavelength increases with frequency.

An LH transmission line with the equivalent circuit from Figure 1a cannot be fabricated as simply as we have suggested here. The inductors and capacitors are inserted into a real hosting environment. Therefore, we have the cell equivalent circuit as shown in Figure 1b. The series impedance and the shunt admittance are now $Z_s = \mathrm{j}\omega L_R + 1/(\mathrm{j}\,\omega C_L)$, $Y_p = \mathrm{j}\omega C_R + 1/(\mathrm{j}\,\omega L_L)$, and we have

$$(3)\quad \beta = \sqrt{\omega^2 L_R C_R + \frac{1}{\omega^2 L_L C_L} - \left(\frac{L_R}{L_L} + \frac{C_R}{L_L}\right)}$$

When the cell length is not infinitesimally short and/or comparable to the wavelength, formula (3) is not valid. In this case the line must be treated as a periodic structure. The equivalent circuit from Figure 1b is now modified to the circuit shown in Figure 2. The hosting line with characteristic impedance Z_0 and phase constant k is here divided into two parts of length $d/2$. To derive the dispersion characteristic of a periodic structure the transmission matrix of the cell is calculated according to [5], and after applying the Floquet theorem [5] we get

$$(4)\quad cos(bd) = cos(kd) - \frac{d^2}{2w^2 L_L C_L} cos^2\left(\frac{kd}{2}\right) + \frac{d}{2} sin(kd) \left(\frac{l}{wC_L Z_0} + \frac{Z_0}{wL_L}\right)$$

$$\left(\frac{1}{\omega C_L Z_0} + \frac{Z_0}{\omega L_L}\right)$$

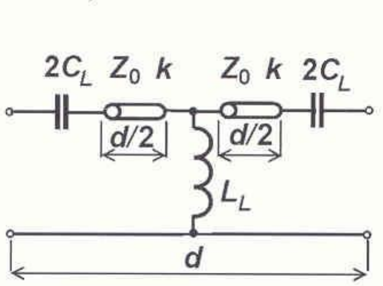

Fig. 2. The equivalent circuit from Figure 1b modified to determine the dispersion characteristic of the periodic structure (4).

In the case of a general periodic structure we can determine the dispersion characteristic numerically by applying, e.g., the CST Microwave Studio electromagnetic simulator in the following way [6]. The elementary cell of a line of length d is terminated at both the input and output faces by periodical boundaries with varying mutual phase shift φ. The resonant frequencies of this cell, which represents a resonator, are calculated by the CST Microwave Studio eigen-mode solver in dependence on φ. This phase shift determines the phase constant

$$(5)\quad \beta = \frac{\varphi}{d}$$

In this way we get the function inverse to the dispersion characteristic, i.e., values of frequency at the chosen phase constant.

III. LH parallel strips

We document the behavior of an LH transmission line on parallel strips. This line is shown in Figure 3. One of the strips is periodically cut by slots representing the series capacitors, and the line is loaded by shunt pins representing the shunt inductors. On its sides, the line is terminated by perfect magnetic walls. Thus we have the line with a unit cell of the form shown in Figure 1b. The dielectric between the strips has permittivity $\varepsilon_r = 2.6$, is $t = 2$ mm in thickness and $w = 10$ mm in width. The length of one cell is $d = 4$ mm, the slot width is 0.05 mm and the diameter of the inductive pins is 0.2 mm. The parameters of the hosting line are [4]

$$(6)\quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}\frac{t}{w}$$

$$(7)\quad k = \frac{\omega}{c}\sqrt{\varepsilon_r}$$

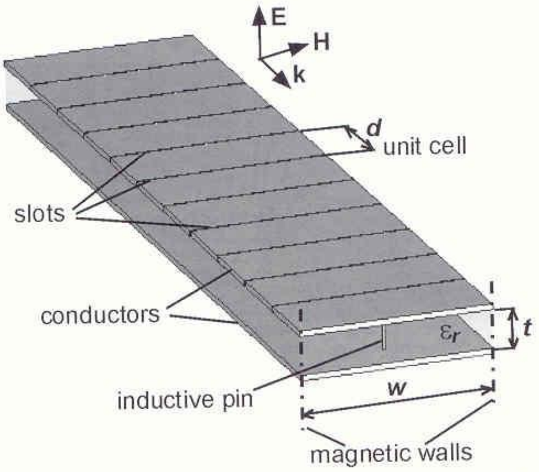

Fig. 3. LH parallel strips.

We get L_R = 1.02 nH, C_R = 0.47 pF. The values L_L and C_R can be calculated from zeros of the phase constant (3). These zeros are coincident with the edges of the stop-band calculated by the CST Microwave Studio, Figure 4. Finally we have C_L = 1.38 pF and L_L = 1.4 nH. The dispersion characteristics of the LH parallel strips computed by CST Microwave Studio, calculated as a periodic structure by (4) and calculated as a homogeneous transmission line by (3), are compared in Figure 4. For low values of the phase constant these three characteristics are nearly identical. For higher values of β the length of the cell is not negligibly short compared to the wavelength, and the line model (3) is not valid. In addition, the phase constant (3) grows to infinity, since, in contrast to the two other models, the model of the homogeneous transmission line does not include the Brag reflection at $\beta d = \pi$.

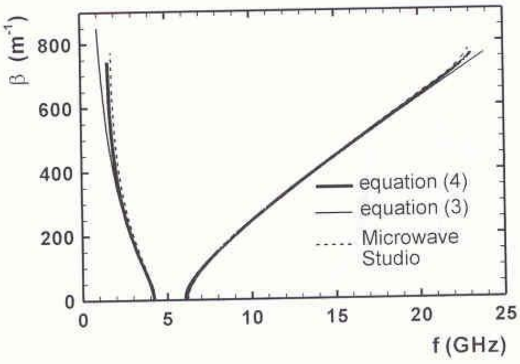

Fig. 4. The dispersion characteristics of the LH parallel strips from Figure 3 calculated by the CST Microwave Studio, and by (3) and (4).

Two lowest pass-bands of the line are shown in Figure 4. The LH wave propagates in the first pass-band from about 2 GHz to 4 GHz. The standard right-handed (RH) wave propagates in the second pass-band from about 6 to 23 GHz. There is a stop-band between these two pass-bands and a stop-band below the LH pass-band due to the high-pass character of the line.

IV. LH volume metamaterial

The structure of the proposed metamaterial utilizes the concept of LH parallel strips proposed above. The modification of this line into a LH volume medium is obvious if we compare Figure 3 with Figure 5. The space between the conductors of the LH parallel strips filled by the dielectric, Figure 3, is transformed into a parallel plate waveguide constituted by the two horizontal conducting planes, Figure 5. The shunt inductors are represented by shunt pins in both cases. In the LH volume medium the series capacitors are represented by the input impedances of the short circuited stubs of the parallel strips.

A CST Microwave Studio model of the unit cell of the proposed volume metamaterial is shown in Figure 5, where the geometrical dimensions are defined. The wave propagates between the two parallel metal strips a in width and $p + 2v$ in length separated by a distance of $2s$. There is a shunt pin of diameter $2r$ representing the shunt inductor at the center of the strip segments. The segment of the parallel strips is terminated by a junction where the two parallel strip short circuited stubs are connected to the line. Their length and height are h-s and p and their input impedance represents a

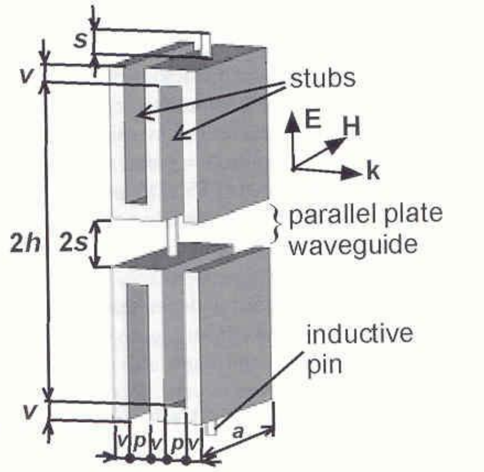

Fig. 5. A unit cell of the proposed bulk left-handed structure.

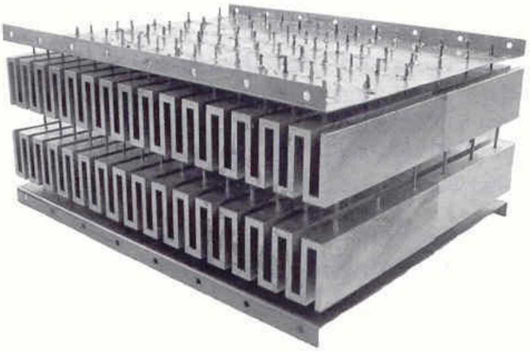

Fig. 6. Manufactured bulk metamaterial consisting of six parallel rows, each of 15 cells, from Figure 5.

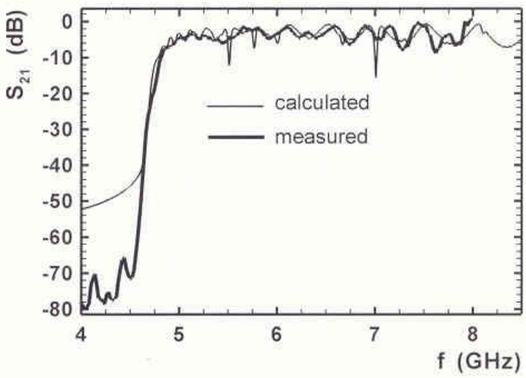

Fig. 7. S_{21} of the metamaterial from Figure 6.

series capacitor. The period of the structure in the direction of the wave propagation is $d = 2p + 2v$. This motif of the cell containing the segment of parallel strips with the junction and two stubs is repeated in all three directions to create a volume structure. Figure 5 shows the two central cells and the two neighbouring cells placed in the vertical direction. They are longitudinally shifted by one half of the period to fill the space completely. These cells are terminated by electric walls at their horizontal planes of symmetry. The incoming wave is thus incident to the matrix of the apertures of the particular parallel plate waveguides. The electric field vector of the propagating wave must be oriented, as shown in Figure 5, across these parallel plate waveguides.

The manufactured bulk LH metamaterial shown in Figure 6 was milled from aluminium blocks. The background material is air, so $\varepsilon_r = 1$. The inductive pins are made of cylindrical wires set in holes drilled into the aluminium body. The dimensions of the structure were obtained by an optimization performed on the Microwave Studio simulator with the aim to obtain a structure with a wide frequency band of LH wave propagation, and at the same time to achieve minimum insertion losses. The resulting dimensions are $a = 20$ mm, $s = 5$ mm, $h = 24$ mm, $p = 3$ mm, $r = 0.5$ mm, $v = 2$ mm. The structure of 6 x 15 cells in the horizontal plane was investigated with electric walls placed from both sides, not shown in Figure 6, i.e., in a waveguide of rectangular cross section.

Figure 7 shows the frequency dependence of the insertion losses of the structure from Figure 6 calculated by the CST Microwave Studio, and also measured. The agreement of these curves is very good. The medium transmits a wave above 4.75 GHz. The frequency band of the LH wave propagation is apparent from the dispersion characteristic shown in Figure 8, and spans between 4.75 and 5.65 GHz. The dispersion characteristic calculated by the CST Microwave Studio was verified experimentally, Figure 8. The propagation constant was determined from the measured wavelength of the standing wave produced by a short located at the output plane. The frequency band of RH wave propagation is from 5.72 to 11.9 GHz. The bands of LH and RH propagation almost merge, and cannot be distinguished in the transmission characteristic. This metamaterial is thus almost balanced [7].

The negative refraction of the LH wave on the surface of the proposed metamaterial was also simulated by the CST Microwave Studio. The structure from Figure 6 was terminated by a section of this metamaterial in the shape of a wedge in order to simulate the oblique incidence of the LH wave to the interface between the metamaterial and air. This wedge was modeled by step by step reduction of the width of each subsequent row by one cell. In this way we get a wedge with an angle of 64° towards the longitudinal axis, Figure 9. The electric field distribution calculated by the CST Microwave Studio at a frequency of 5 GHz showing

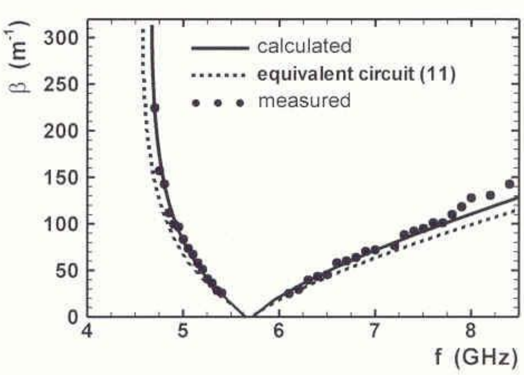

Fig. 8. The dispersion characteristic of the metamaterial from Figure 6.

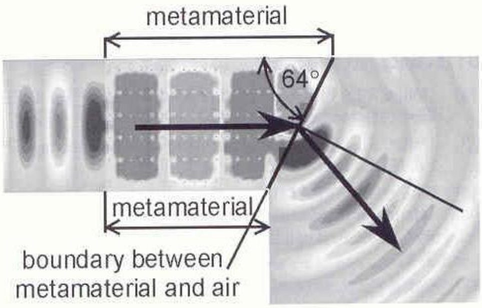

Fig. 9. The distribution of an electric field on the horizontal plane crossing the inductive pins at a central waveguiding part calculated at 5 GHz.

the negative refraction on the boundary between the metamaterial and air is plotted in Figure 9.

V. Equivalent circuit

The equivalent circuit of one cell of the studied metamaterial is shown in Figure 2. The total length of the cell, which defines the period of the metamaterial, is $d = 2p + 2v$, Figure 5. The circuit consists of the hosting parallel strips divided into two parts $l/2 = p/2 + v$ in length, a series capacitor with capacity C_L, and a shunt inductor with inductance L_L. By properly replacing the variables in (6) we get the characteristic impedance of the hosting line

$$(8) \quad Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \frac{2s}{a}$$

Phase constant k is determined by (7) now with $\varepsilon_r = 1$. Series capacitance C_L is the input capacitance of the short circuited stub $l_S = h\text{-}s$ in length and p in height [4]

$$(9) \quad C_L = -\frac{1}{2\pi f \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} \frac{p}{a} tan\left[k(h-s)\right]}$$

The cylindrical pin of length $2s$ and radius r represents shunt inductance L_L [8]

$$(10) \quad L_L = 2 \cdot 10^{-7} \cdot 2s\, ln\left(\frac{2s}{r\sqrt{\pi}}\right) + 0.5 + 0.2235 \frac{r\sqrt{\pi}}{s}$$

By properly modifying (4), we obtain the dispersion characteristic of the infinite cascade of cells from Figure 2

$$(11) \quad cos(\beta d) = cos(kl') + \frac{l'^2 Z_s Y_p}{2} cos^2\left(k\frac{l'}{2}\right) + \frac{jl'}{2}\left(\frac{Z_s}{Z_0} + \frac{Y_p}{Y_0}\right) sin(kl')$$

where Z_0 and Y_0 are the characteristic impedance and admittance of the hosting parallel strips (8), $Z_s = 1/(j\omega C_L)$ and $Y_p = 1/(j\omega L_L)$ are the series impedance and the shunt admittance of the cell. l' is the modified length of the hosting parallel strips in excess of physical length l. The excess length l' is responsible for the reactance of the junction between the parallel strips and the stubs. Fitting the dispersion characteristic (11) to the same characteristic calculated by the CST Microwave Studio resulted in $l' = l + 0.6p$. Figure 8 shows the good accord between the propagation constant calculated by means of (11) and by the CST Microwave Studio, and the measured characteristic, except at high frequencies in the RH region. The propagation constant calculated according to (11) differs at high frequencies from the two remaining values, since the equivalent circuit gradually loses its validity with increasing frequency.

VI. Conclusions

This paper summarizes the results of an investigation of LH media composed of periodic structures. Each cell of the periodic structure is represented by a segment of the hosting line together with a series capacitor and a shunt inductor. These media were treated both as a periodic structure and as a continuous transmission line. The latter is valid when the cell length is much shorter than the wavelength. To validate this concept the LH parallel strips were investigated.

A new volume metamaterial is proposed. Its structure incorporates the matrix of the left-handed transmission lines repeated both horizontally and vertically. These lines consist of parallel strip segments shunted at their centers by inductive pins representing shunt inductors. The segments are separated by short circuited parallel strip stubs representing series capacitors. The metamaterial exhibits left-handed behaviour in the frequency band from 4.75 to 5.65 GHz. The transmission and dispersion characteristics of this medium were calculated and measured. They fit very well with each other. In addition, the equivalent circuit of the metamaterial was proposed. The new medium offers LH propagation in a wider frequency band than do metamaterials with split-ring resonators. It is a suitable candidate for applications when propagation of an LH wave is required, e.g, phase compensation, resonators, special waveguides, ideal lenses, etc. The structure is able to carry an LH high power wave.

Acknowledgments

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