Homogenization of Fishnet Metamaterials: Possibilities and Restrictions

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Abstract

The possibilities and restrictions of homogenization in fishnet metamaterials are studied. By means of 1D and 2D dispersion diagrams obtained by combination of mode-matching with generalized eigenvalue problem it is shown that in narrow frequency region just below Wood's anomaly the approximation of fishnet by uniaxial medium is allowed. It is also shown that this is the frequency band of possible negative refraction.

1. Introduction

Fishnet metamaterials made of a stack of two or more metallic plates perforated by small holes have been extensively studied during recent years [1] - [7]. In these works, negative refractive index, extracted from the analysis of the transmission an reflection for normally incident plane waves, as well as backward-wave propagation inside the stack, were extensively studied both theoretically and experimentally. Recently, negative refraction at the exit interface of a prism made of a fishnet metamaterial has been shown experimentally at microwaves [8] and optical frequencies [9]. All these theoretical analysis and experimental evidences, together with the simplicity of the design, are the basis of the present interest on this kind of metamaterial structures.

The aim of this contribution is to show the conditions for negative and positive refraction at the fishnet interface and the possibilities and limits for the homogenization of the structure.

2. Analysis and Results

The sketch of the unit cell of the analyzed structure is shown in Fig.1a,b together with wave markings on each boundary. The analysis is based on mode-matching solution of scattering problem of waves a_3, b_3, a_4, b_4 (see Fig. 1b) and on corresponding generalized eigenvalue problem for longitudinal wavenumber q_z. Lateral wavenumbers q_x and q_y are parameters of the calculation. The eigen-analysis is made for two particular polarizations. First one is called TE and it is characterized by wavenumbers $q_x \neq 0, q_y = 0, q_z \neq 0$ and electric y-sidewalls. Second polarization will be called TM having $q_x = 0, q_y \neq 0, q_z \neq 0$ and magnetic x-sidewalls. The TE polarization corresponds to the planewave with field components E_y, H_x, H_z impinging on the fishnet, while the TM polarization corresponds to the planewave with components E_y, E_z, H_x.

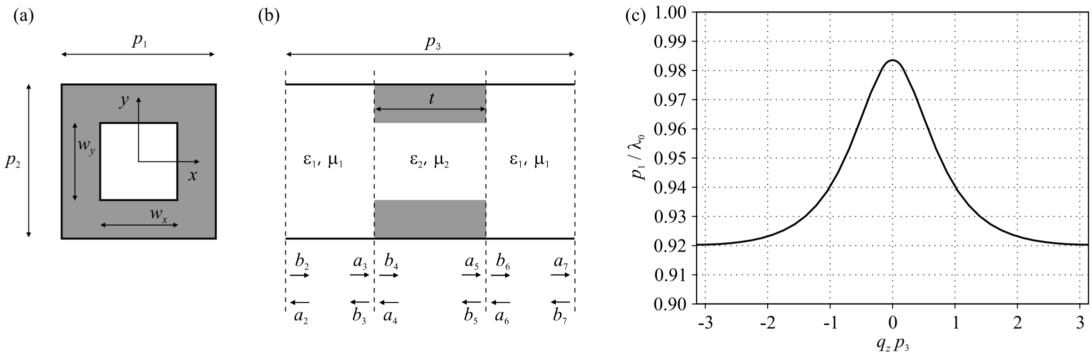

Fig. 1: Panels (a) and (b) show front and side view of the analyzed unit cell. The longitudinal direction is treated as z-direction. The structure is described by the dimensions of rectangular window w_x, w_y, by lateral periodicities p_1, p_2, by longitudinal periodicity p_3 and by diaphragm thickness t. Panel (c) shows dispersion diagram for normal incidence in the structure with $w_x = w_y = p_1/3$, $p_3 = p1/9$, $t = p_1/30$ and vacuum dielectrics.

For normal incidence the fishnet structure exhibits left-handed passband just below Wood's anomaly. This passband is depicted in Fig. 1c for particular structural parameters. There is no mode able to propagate at lower frequencies, so there is a stopband below the left-handed band shown in Fig. 1c spanned down to zero frequency. For higher frequencies the propagation can be multimodal and thus not consistent with effective medium approximation. For oblique incidence it can be shown that TE case leads to high dispersion and thus high values of q_z except of the close vicinity of Wood's anomaly. The region of possible homogenization can then only be located in narrow frequency band just below Wood's anomaly.

To see the fishnet dispersion in more details, the iso-frequency contours in the above mentioned frequency region are for both polarisations shown in Fig. 2. Since we are interested in the homogenization, the allowed wavenumbers are restricted by

$$|q_x \cdot p_1|^2 + |q_y \cdot p_2|^2 + |q_z \cdot p_3|^2 < (\pi/5)^2 . \tag{1}$$

Fig. 2 shows that Eq. (1) restricts the frequency band of possible homogenization considerably. On the other hand there exists narrow frequency region where the iso-frequency contours are conic sections, particularly hyperbolas for TE case and ellipses for TM case, suggesting that in this frequency region the fishnet can be seen as an uniaxial medium characterized by permittivity and permeability $\epsilon_{r\perp}$, $\mu_{r\perp}$ in transversal direction and ϵ_{rz}, μ_{rz} in longitudinal direction and described by dispersion relation

$$q_z^2 = \varepsilon_{r\perp}\mu_{r\perp}k_0^2 - \frac{\mu_{r\perp}}{\mu_{rz}}q_x^2 \tag{2}$$

for TE case and

$$q_z^2 = \varepsilon_{r\perp}\mu_{r\perp}k_0^2 - \frac{\varepsilon_{r\perp}}{\varepsilon_{rz}}q_y^2 \tag{3}$$

for TM case. Such fitting can really be done with negligible error, allowing us to conclude that there is a frequency region in which the homogenization of the fishnet is possible. The dispersion contours of Fig. 2 then show that on the hypothetical boundary between vacuum and fishnet the TM polarized wave would exhibit "negative" refraction, while the TE polarized wave would exhibit "positive" refraction which is coherent with previous experimental observations [8].

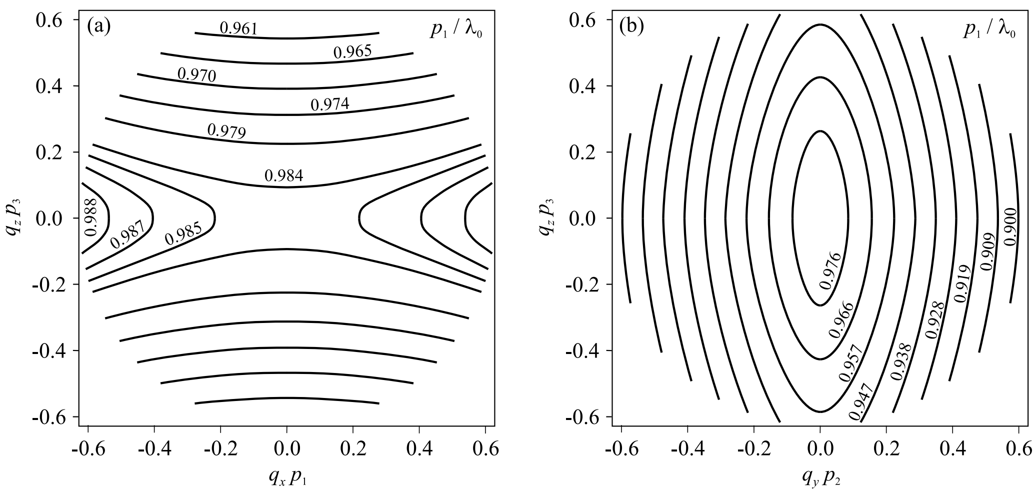

Fig. 2: Isofrequency contours for (a) TE polarization (b) TM polarization. The structural parameters are identical with Fig. 1.

3. Conclusion

It has been shown that index of refraction cannot be safely defined in fishnet metamaterials. On the other hand, the homogenization is not completely forbidden and, in narrow frequency band below Wood's anomaly, the fishnet can be seen as uniaxial medium and can exhibit negative refraction for some polarization of incident planewave.

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